ges) **Reg. No. :** ....

ue No. : 5315 **Sub. Code : PMAM 22/ ZMAM 22**

:. (CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics — Core

ANALYSIS — II

(For those who joined in July 2017 onwards)

: Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

Let $P = \{0,\ 0.2,\ 0.8,\ 0.9,\ 1\}$ be a partition of $[0,1]$ which one of the following is refinement of P

(a) $\{0,\ 0.2,\ 0.7,\ 0.9,\ 1\}$

(b) $\{0,\ 0.2,\ 0.6,\ 0.8,\ 0.93,\ 1\}$

(c) $\{0,\ 0.2,\ 0.3,\ 0.4,\ 0.8,\ 0.9,\ 1\}$

(d) $\{0,\ 0.2,\ 0.6,\ 0.7,\ 0.8,\ 1\}$

2. $f \in R(\alpha)$ on $[a,b]$ if and only if for every $\varepsilon > 0$

(a) $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$ for every partition P

(b) there exists a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$

(c) there exists a partition P such that $L(P,f,\alpha) - U(P,f,\alpha) < \varepsilon$

(d) $L(P,f,\alpha) - U(P,f,\alpha) < \varepsilon$ for every partition P

3. Let $s_{m,n} = \dfrac{m}{m+n}$; $\begin{matrix} m = 1,2,3,.. \\ n = 1,2,3,.. \end{matrix}$, then $\lim\limits_{m\to\infty} \lim\limits_{n\to\infty} s_{m,n}$ is

(a) 1 (b) $\dfrac{1}{2}$

(c) ∞ (d) 0

4. A sequence $\{f_n\}$ converges to f w.r.t. the metric of $\mathscr{C}(X)$ if and only if

(a) $f_n \to f$ on X

(b) f is continuous on X

(c) $f_n \to f$ uniformly on X

(d) f_n and f are continuous on X

If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3.,,,$ and if $\{f_n\}$ is ———— and ———— on K then $\{f_n\}$ is uniformly bounded on K.

(a) pointwise bounded and continuous

(b) pointwise bounded and equicontinuous

(c) continuous and differentiable

(d) a sequence of continuous and bounded functions

Let $f_n(x) = \dfrac{x^2}{x^2 + (1-nx)^2}$ $(0 \le x \le 1,\ n = 1,2,3,...)$ then $f_8(1/8)$ is

(a) 0 (b) 1

(c) ∞ (d) 8

The set of continuous functions on $[a,b]$ is the uniform closure of the set of polynomials on $[a,b]$. This statement is known as

(a) Lagrange theorem

(b) Weierstrass theorem

(c) Stone's theorem

(d) Cauchy's theorem

8. $\lim\limits_{n\to\infty}\left(1 + \dfrac{x}{n}\right)^n = ?$

(a) e^x (b) e^n

(c) e (d) 1

9. The Dirichlet Kernel $D_N(x)$ is also equal to

(a) $\dfrac{\sin(N+1)x}{\sin x}$ (b) $\dfrac{\sin(N+1/2)x}{\sin(x/2)}$

(c) $\dfrac{\sin(N+1/2)x}{\sin x}$ (d) $\dfrac{\sin(N+1)x}{\sin(x/2)}$

10. The value of $\Gamma(1/2)$ is

(a) 1/2 (b) 1

(c) π (d) $\sqrt{\pi}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If P^* is a refinement of P, prove that $L(P, f, \alpha) \le L(P^*, f, \alpha)$.

Or

(b) If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$, prove that $f \in R(\alpha)$.

12. (a) Give an example to show that a convergent series of continuous functions may have a discontinuous functions sum.

Or

(b) State and prove the Weierstrass test for uniform convergence.

13. (a) Let α be monotonically increasing on $[a, b]$ suppose $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$ for $n = 1, 2, 3,...$ and suppose $f_n \to f$ uniformly on $[a, b]$. Prove that $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$ and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.

Or

(b) If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3...$ and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ is uniformly bounded on K.

14. (a) Define an algebra and the uniform closure of an algebra. Let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of bounded functions. Prove that $\mathscr{B}$ is a uniformly closed algebra.

Or

(b) Suppose Σc_n converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n (-1 < x < 1)$ prove that $\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} c_n$.

15. (a) Define the Dirichlet Kernel $D_n(x)$ and show that $D_N(x) = \dfrac{\sin(N+1/2)^x}{\sin(x/2)}$ and $s_N(f; x) = \dfrac{1}{2\pi} \int_{-\pi}^{\pi} f(x-t) D_N(t)\, dt$.

Or

(b) If $x > 0$ and $y > 0$, prove that $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \dfrac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $f_1, f_2 \in \mathscr{R}(\alpha)$ on $[a,b]$, prove that $f_1 + f_2$, $cf_1 \in \mathscr{R}(\alpha)$ for every constant c and $\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$, $\int_a^b cf_1 d\alpha = c\int_a^b f_1 d\alpha$.

Or

(b) Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$ and show that $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\, dx$.

17. (a) If γ' is continuous on $[a, b]$, prove that γ is rectifiable and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

Or

(b) Suppose $f_n \to f$ uniformly on a set E in a metric space. Prove that $\lim_{t\to x} \lim_{n\to\infty} f_n(t) = \lim_{n\to\alpha} \lim_{t\to x} f_n(t)$.

18. (a) Prove that the exists a real continuous function on the real line which is nowhere differentaiable.

Or

(b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set E prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in F$.

19. (a) If f is a continuous complex function on $[a, b]$, prove that there exists a sequence of polynomials p_n such that $\lim_{n\to\alpha} P_n(x) = f(x)$.

Or

(b) State and prove Taylor's theorem.

20. (a) Suppose $a_0, a_1, .., a_n$ are complex numbers, $n \geq 1$, $a_n \neq 0$. $P(z) = \sum_0^n a_k z^k$. Prove that $P(z) = 0$ for some complex numberz.

Or

(b) If f is a positive function on $(, \infty)$ such that (i) $f(x+1) = xf(x)$ (ii) $f(1) = 1$ (iii) $\log f$ is convex, prove that $f(x) = \Gamma(x)$.